Suite 6.4.2

$$f_{a,b}\left(x+\frac{i\pi}{2}\right) = -i\,\frac{\exp\left(ax+\frac{i\pi a}{2}\right)-\exp\left(bx+\frac{i\pi b}{2}\right)}{ch(x)}$$

$$f_{a,b}\left(x-\frac{i\pi}{2}\right) = i\,\frac{\exp\left(ax-\frac{i\pi a}{2}\right)-\exp\left(bx-\frac{i\pi b}{2}\right)}{ch(x)}$$

On a donc par le théorème des résidus on a :

$$J_{a,b} = 2\pi i\,\mathrm{Res}(f_{a,b}, 0) \qquad sh(z) = sh(0) + z\,\underbrace{ch(0)}_{1} + o(z^2)$$

$$= 2\pi i \lim_{z\to 0} \frac{\exp(az)-\exp(bz)}{(z)+o(z^2)} = 0$$

$$\Rightarrow J_{a,b} = -i\int_{-\infty}^{+\infty} \frac{\exp\left(ax+\frac{i\pi a}{2}\right)-\exp\left(bx+\frac{i\pi b}{2}\right)}{ch(x)}\,dx$$

$$+i\int_{-\infty}^{\infty} \frac{\exp\left(ax-\frac{i\pi a}{2}\right)-\exp\left(bx-\frac{i\pi b}{2}\right)}{ch(x)}\,dx$$

$$= \left(-i\exp\left(\frac{i\pi a}{2}\right)+i\exp\left(-\frac{i\pi a}{2}\right)\right)\int_{-\infty}^{\infty}\frac{\exp(ax)}{ch(x)}\,dx$$

$$\left(+i\exp\left(\frac{i\pi b}{2}\right)-i\exp\left(-\frac{i\pi a}{2}\right)\right)\int_{-\infty}^{\infty}\frac{\exp(bx)}{ch(x)}\,dx$$

$$= -i\,2i\,\sin\left(\frac{\pi a}{2}\right)\int_{-\infty}^{\infty}\frac{\exp(ax)}{ch(x)}\,dx + i\,2i\,\sin\left(\frac{\pi b}{2}\right)\int_{-\infty}^{\infty}\frac{\exp(bx)}{ch(x)}\,dx$$

$$= \sin\left(\frac{\pi a}{2}\right)\frac{\pi}{\sin\left(\frac{\pi a}{2}\right)} - \sin\left(\frac{\pi b}{2}\right)\frac{\pi}{\sin\left(\frac{\pi b}{2}\right)} = 0$$

Ex 6.5

Ex 6.5.1 $\quad I_\alpha = \int_{-\infty}^{\infty} \frac{x^\alpha}{(1+x)^n}\,dx$

On pose $J_\alpha = \int_{\gamma_R} \frac{z^\alpha}{(1+z)^n}\,dz \qquad z^\alpha = e^{\ln(z^\alpha)} = e^{\alpha \ln(z)}$

$$\frac{d^n}{dz^n}(z^\alpha) =$$

Ex 6.5.2

$$I_\alpha = \int_{-\infty}^{\infty} \frac{x^\alpha}{1+x^2}\,dx \qquad Cv \Rightarrow \alpha < 1$$

$$J_\alpha = \int_{\gamma_R} \frac{z^\alpha}{1+z^2}\,dx \qquad z^\alpha = e^{\ln(z^\alpha)} = e^{\alpha\ln(z)}$$